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Josephson junction dynamics in the presence of microresistors and an AC drive

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Abstract. The motion of fluxons in a long Josephson junction with point-like inhomogeneities (microresistors) and a dissipative loss is studied in a semi-analytical and fully numerical manner. It is found that the dynamic behaviour of fluxons is markedly changed by the combined effect of microresistors and an AC drive, i.e. the AC drive allows, via the microresistors, the fluxons to overcome a dissipative resistance and to move forward linearly. A physical interpretation of the significant novel phenomenon is given. First, the microresistors generate a new type of inherent localized structure, the so-called impurity mode. Then, the impurity modes are excited and amplified by the AC drive. Finally, the fluxon gains, through the strong fluxon–impurity mode (microresistor) interactions, kinetic energy to overcome the attractive potentials of the microresistors and the dissipative resistance, and it escapes from the microresistors eventually.

1. Introduction

The dynamics of long Josephson junctions (LJJs) have been a very active and fruitful area of research for many years. In particular, since the recent discovery of high-T_c superconductivity [1] there has been renewed interest. In addition, the LJJs are very interesting objects for observing various dynamical non-linear effects including soliton propagation and chaotic behaviour [2]. Recently, the successful production of LJJs with microresistors [3] has aroused increasing interest in order to study the propagation of fluxons in the microresistors. In particular, recent developments in non-linear phenomena in inhomogeneous systems [4, 5] have provided valuable new insight into the dynamics of LJJs with microresistors.

The dynamics of a LJJ are governed by the perturbed sine–Gordon (SG) equation [6]

$$\Phi_{tt} - \Phi_{xx} + \sin\Phi = -\alpha\Phi_t + \sum_{i=1}^{n} \varepsilon_i \delta(x - l_i) \sin\Phi + \eta_{\mathrm{DC}} + \eta_{\mathrm{AC}}. \qquad (1)$$

Here Φ is the quantum-mechanical phase difference between the two films. The spatial variable x is measured in units of penetration depth λ_j and time t in units of the reciprocal plasma frequency ω_p^{-1}. $\alpha\Phi_t$ represents the dissipation due to tunnelling of normal electrons across the barrier, ε_i is the 'strength' of the ith microresistor ($\varepsilon_i > 0$). η_{DC} and η_{AC}, are the distributed DC and AC bias currents, respectively, providing energy input. The AC bias current is given by

$$\eta_{\mathrm{AC}} = -A\sin(\omega t).$$

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A fluxon is described by a kink solution of the unperturbed SG equation as follows:

$$\Phi_k(x,t) = 4\tan^{-1}\left[\exp\left(\sigma\frac{x - X(t)}{\sqrt{1-V^2}}\right)\right] \tag{2}$$

where $V(V^2 \leqslant 1)$, $X(t) = Vt$ and $\sigma = \pm 1$ are the kink's velocity, centre-of-mass coordinate and polarity, respectively.

For many years, the dynamics of LJJs with localized separate microresistors have been studied following the collective-coordinate method of MacLaughlin and Scott [6] and much significant information has been obtained [6–8]. In the theory of MacLaughlin and Scott, a microresistor (impurity) produces a potential hill of a height U_0. The external drives η_{AC} and η_{DC} result from alternate and constant forces, respectively, exerted on the fluxon. In the absence of the DC bias current and the dissipative resistance, a critical velocity of the fluxon occurs. The fluxon with a velocity less than the critical velocity may be trapped by microresistors. In the presence of the DC bias current and the dissipative resistance, the equilibrium velocity V_0 of the fluxon is, when $\eta_{DC}^2 > \alpha^2 U_0$, determined by the balance between the DC-driven force and the friction [6–8]:

$$V_0 = -(\pi\sigma\eta_{DC}/4\alpha)[1 + (\pi\eta_{DC}/4\alpha)^2]^{1/2}. \tag{3}$$

The fluxon, if $\eta_{DC}^2 < \alpha^2 U_0$, slows gradually and is finally trapped by microresistors. In the opinion of MacLaughlin and Scott, an AC drive results from the alternate force of time t and acts directly on the fluxon. So in an AC drive the trapped fluxon oscillates in the vicinity of the microresistor. The oscillating amplitude is of the same order of the magnitude as the amplitude of the AC drive. Therefore, when the AC drive is weak, one naturally expects the effect introduced by the AC drive to be negligible. So, most previous workers [6–8] used the model of a damped DC-driven LJJ with localized microresistors for studying the LJJ dynamics.

However, more recent research on the dynamics in a non-linear inhomogeneous system has revealed some novel features [4, 5, 9, 10]. Firstly, the microresistors in harmonic lattices generate a new type of inherent localized structure, the so-called impurity mode [10], i.e. local oscillations around the impurities with some inherent frequencies [4, 5, 10]. Then, the localized impurity modes can be excited and amplified by an AC drive. Finally, the motion of the fluxon, through interactions between the fluxon and excited and amplified localized impurity modes, might be essentially changed [4, 5]. In the present paper we shall show that, because a microresistor ($\varepsilon > 0$) supports an inherent localized structure, the AC drive does not necessarily act directly on a fluxon but excites impurity modes. Therefore, the microresistors in the AC drive, through fluxon–microresistor interaction, can essentially change the motion of the fluxon, i.e. in the absence of a DC drive the fluxon may overcome a strong dissipative resistance and move forward linearly.

This paper is organized as follows. In section 2 we firstly, according to perturbation theory [4, 5], describe theoretically the fluxon–microresistor interactions in the limit of weak AC drive and weak dissipative resistance and in the presence of two microresistors; we then numerically study the interactions. In section 3 we numerically analyse the fluxon–microresistor interactions in an AC drive, a strong dissipative resistance and many separate microresistors. Concluding remarks are contained in section 4.

2. Perturbation theory

In this paper, we consider equation (1) with $\eta_{DC} = 0$ in great detail. Therefore, the basic equation in the present paper is

$$\Phi_{tt} - \Phi_{xx} + \sin\Phi = -\alpha\Phi_t + \sum_{i=1}^{n} \varepsilon_i \delta(x - l_i) \sin\Phi + \eta_{AC}. \tag{1a}$$

In the case of weak AC drive and weak dissipative resistance, we only discuss the special case of two separate microresistors and $\varepsilon_1 = \varepsilon_2 = \varepsilon$, because it can display the main qualitative features of our problem. It should be noted that, even if the perturbation is non-zero but is small, equation (1) still has a kink-type solution. T_Q describes fluxon–microresistor interactions in a weak dissipative resistance and a weak AC drive; firstly we treat the fluxon as a quasi-particle moving in an inhomogeneous system and assume the fluxon position to be the particle coordinates [4, 5]; then we note that the microresistor ($\varepsilon > 0$) supports a localized oscillating state, i.e. the impurity mode [4, 5, 10]. In the case of $\alpha = 0$ and $A = 0$, linearizing equation (1*a*) with respect to small Φ yields the following solution for the impurity mode [10]:

$$\Phi_{\mathrm{im}}(x, t) = a_1(t) \exp(-\varepsilon|x - l_1|/2) + a_2(t) \exp(-\varepsilon|x - l_2|/2) \tag{4}$$

where

$$a_1(t) = a_{10} \cos(\Omega t + \theta_{10}) \tag{5a}$$

$$a_2(t) = a_{20} \cos(\Omega t + \theta_{20}) \tag{5b}$$

$$\Omega = \sqrt{1 - \varepsilon^2/4} \tag{6}$$

is the inherent frequency of the impurity mode, and θ_{10} and θ_{20} are constant phases. Hereafter, we take $\varepsilon = 0.7$, $l_1 = 0$ and $l_2 = l = 4$ in our calculation.

Now, let us return to equation (1*a*) and analyse the fluxon–microresistor interactions by means of the so-called collective-coordinate approach [4, 5] taking three dynamical variables into account and the associated numerical calculation (in brief, the semi-analytical approach [4]). The three dynamical variables are the fluxon position $X(t)$, and the amplitudes $a_1(t)$ and $a_2(t)$ of the impurity-mode oscillations. Assuming that $A \ll 1$ and substituting the *ansatz* $\Phi = \Phi_{\mathrm{k}} + \Phi_{\mathrm{im}}$ into the Lagrangian appropriate to equation (1*a*) [4, 5], i.e.

$$L = \int_{-\infty}^{+\infty} \mathrm{d}x \bigg(\frac{\Phi_t^2}{2} - \frac{\Phi_x^2}{2} - [1 - \varepsilon\delta(x) - \varepsilon\delta(x - l)](1 - \cos\Phi) \\ - A(\Phi - \pi)\sin(\omega t) + L_{\mathrm{f}} \bigg) \tag{7}$$

where

$$L_{\mathrm{f}} = \tfrac{1}{2} \int_0^t \Phi_{t'}^2(x, t')\, \mathrm{d}t' \tag{8}$$

in the lowest-order approximation and weak relativistic case ($V^2 \ll 1$), we can derive the following approximate Lagrangian:

$$L = 4\dot{X} + \varepsilon^{-1}(\dot{a}_1^2 - \Omega^2 a_1^2) + \varepsilon^{-1}(\dot{a}_2^2 - \Omega^2 a_2^2) + (1 + 2\varepsilon^{-1}) \exp(-\tfrac{1}{2}\varepsilon l)\dot{a}_1\dot{a}_2 - \tfrac{1}{4}\varepsilon^2(2\varepsilon^{-1} - l) \\ \times \exp(-\tfrac{1}{2}\varepsilon l)\, a_1 a_2 - U(X) - a_1 F(X) - U(X - l) - a_2 F(X - l) \\ - 4A\varepsilon^{-1} a_1 \sin(\omega t) - 4A\varepsilon^{-1} a_2 \sin(\omega t) + 2\pi A X \sin(\omega t) + \bar{L}_{\mathrm{f}} \tag{9}$$

where

$$\bar{L}_f = \int_0^t [4\alpha\dot{X}(t') + \alpha\varepsilon^{-1}\dot{a}_1^2(t') + \alpha\varepsilon^{-1}\dot{a}_2^2(t') + \alpha(1+2\varepsilon^{-1})\exp(-\tfrac{1}{2}\varepsilon l)\,\dot{a}_1(t')\dot{a}_2(t')]\,\mathrm{d}t' \tag{10a}$$

$$U(X) = -\frac{2\varepsilon}{\cosh^2 X} \tag{10b}$$

$$F(X) = -\frac{2\varepsilon\sinh X}{\cosh^2 X}. \tag{10c}$$

The corresponding equations of motion for the three dynamical variables are

$$8\ddot{X} + 8\alpha\dot{X} + U'(X) + U'(X-l) + a_1(t)F'(X) + a_2(t)F'(X-l) - 2\pi A\sin(\omega t) = 0 \tag{11a}$$

$$\ddot{a}_1(t) + \tfrac{1}{2}\varepsilon^{-1}(1+2\varepsilon^{-1})\exp(-\tfrac{1}{2}\varepsilon l)\,\ddot{a}_2(t) + \alpha\dot{a}_1(t) + \tfrac{1}{2}\alpha\varepsilon(1+2\varepsilon^{-1})\exp(-\tfrac{1}{2}\varepsilon l)\,\dot{a}_2(t) + \Omega^2 a_1(t) + \tfrac{1}{8}\varepsilon^3(2\varepsilon^{-1} - l)\exp(-\tfrac{1}{2}\varepsilon l)\,a_2(t) + \tfrac{1}{2}\varepsilon F(X) + 2A\sin(\omega t) = 0 \tag{11b}$$

$$\ddot{a}_2(t) + \tfrac{1}{2}\varepsilon^{-1}(1+2\varepsilon^{-1})\exp(-\tfrac{1}{2}\varepsilon l)\,\ddot{a}_1(t) + \alpha\dot{a}_2(t) + \tfrac{1}{2}\alpha\varepsilon(1+2\varepsilon^{-1})\exp(-\tfrac{1}{2}\varepsilon l)\,\dot{a}_1(t) + \Omega^2 a_2(t) + \tfrac{1}{8}\varepsilon^3(2\varepsilon^{-1} - l)\exp(-\tfrac{1}{2}\varepsilon l)\,a_1(t) + \tfrac{1}{2}\varepsilon F(X-l) + 2A\sin(\omega t) = 0. \tag{11c}$$

The system (11) describes a quasi-particle with the coordinate $X(t)$ and mass 8, located in the attractive potentials of $U(x)$ and $U(x-l)$, and the AC drive with the amplitude $2\pi A$ and coupled with the harmonic oscillators $a_1(t)$ and $a_2(t)$ which are coupled to each other and excited by the AC drive. From equation (11) we see that the fluxon, owing to the attraction of the microresistors, very slowly approaches the first microresistor and in this process the effect of the AC drive on the fluxon can make the latter oscillate with an amplitude of the same order of magnitude as the amplitude of the AC drive. So, when A is very small, the effect is negligible. However, we observe that the inherent impurity modes, in the presence of the AC drive, can be excited and amplified after a finite time, as shown in figure 1. Then, when the fluxon is close to the excited and amplified impurity modes (microresistors), strong interaction takes place, and hence the motion of fluxon is essentially changed.

Before solving equation (11) numerically, we explain the physical mechanism of fluxon–microresistor interaction by means of an energy exchange between the kink translational mode (fluxon) and the impurity mode (microresistor) [5]. The energy E_{im} stored in an impurity mode

$$\psi_{\mathrm{im}} = a_0\sin(\Omega t + \theta_0)\exp(-\tfrac{1}{2}\varepsilon|x|) \tag{12}$$

may be easily calculated as

$$E_{\mathrm{im}} = \frac{1}{2}\int_{-\infty}^{+\infty}\left[\left(\frac{\partial\psi_{\mathrm{im}}}{\partial t}\right)^2 + \left(\frac{\partial\psi_{\mathrm{im}}}{\partial x}\right)^2 + [1-\varepsilon\delta(x)]\psi_{\mathrm{im}}\right]^2 \mathrm{d}x = \frac{\Omega^2 a_0^2}{\varepsilon}. \tag{13}$$

To calculate the energy an excited impurity mode transfers to a fluxon after a fluxon–microresistor interaction; let us consider a fluxon with an initial velocity $V_i > 0$, coming

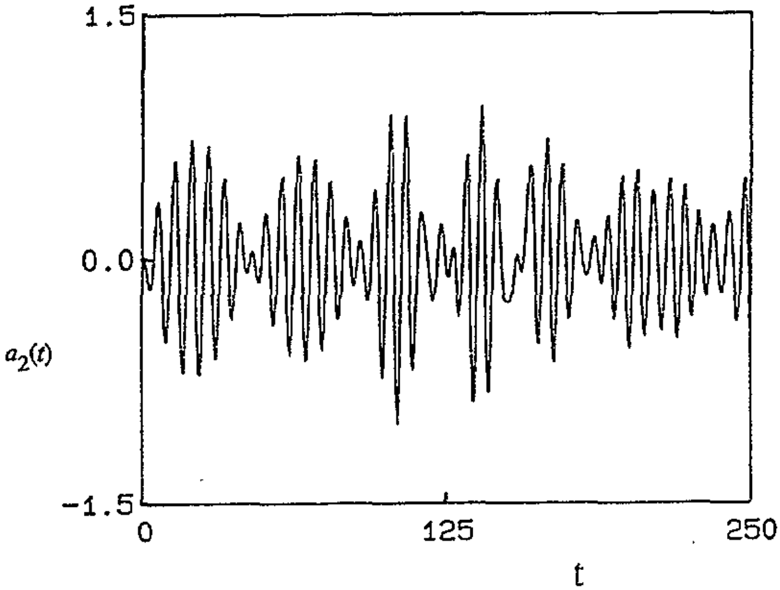

Figure 1. Solution of equation (11): $a_2(t)$ versus time t for parameters $A = 0.065$, $\alpha = 0.013$ and $\varepsilon = 0.7$. The impurity modes are excited and amplified to give $a_2(t)$.

from $-\infty$. Then in the zeroth-order approximation the equation of motion, equation (11a), for the fluxon coordinate $X(t)$ is reduced to the form $8\ddot{X} + U'(X) = 0$, which has an exact solution

$$X(t) = \sinh^{-1}[B\sin(Vt)] \tag{14}$$

where $B = \sqrt{V^2 + \frac{1}{2}\varepsilon}\Big/V$. Now we insert the result (14) into $F(x)$ of the equation of motion, equation (11b), and take the function

$$f(t) = -\frac{\varepsilon}{2}F[X(t)] = \frac{\varepsilon^2 B\sinh(Vt)}{1 + A^2\sinh^2(Vt)} \tag{15}$$

as a pulse force acting on the harmonic oscillator. Introducing the complex variable $\xi(t) = \dot{a}_1 + \mathrm{i}\Omega a_1(t)$ and assuming that the effect of the fluxon–microresistor interaction is much stronger than the dissipative resistance, we find that equation (11b) may be reduced to $\dot{\xi} - \mathrm{i}\Omega\xi = f(t)$, which has the solution

$$\xi(t) = \exp(\mathrm{i}\Omega t)\int_{-\infty}^{t} f(\tau)\exp(-\mathrm{i}\Omega\tau)\,\mathrm{d}\tau + \Omega a_0\exp(\mathrm{i}(\Omega t + \theta_0)) \tag{16}$$

with the initial conditions

$$a_1(t)|_{t\to-\infty} = a_0\sin(\Omega t + \theta_0) \tag{17a}$$

$$\dot{a}_1(t)|_{t\to-\infty} = a_0\cos(\Omega t + \theta_0) \tag{17b}$$

which indicate that the oscillator (impurity mode) is excited prior to the fluxon–microresistor interaction. The total energy stored in the impurity mode after the fluxon–microresistor interaction easily calculated to be

$$\overline{E_{\mathrm{im}}}(V_{\mathrm{i}}, a_0, \theta_0) = \frac{1}{\varepsilon}|\xi(+\infty)|^2 = \frac{1}{\varepsilon}\left|\int_{-\infty}^{+\infty} f(t)\exp(-\mathrm{i}\Omega t)\,\mathrm{d}t + \Omega a_0 \exp(\mathrm{i}\theta_0)\right|^2$$

$$= \frac{1}{\varepsilon}[G^2(V_{\mathrm{i}}) + \Omega^2 a_0^2 + 2G(V_{\mathrm{i}})a_0\Omega\cos\theta_0] \tag{18}$$

where V_{i} is the fluxon initial velocity, a_0 is the initial amplitude of the impurity mode, θ_0 is the phase of the impurity oscillation at the collision-instant,

$$G(V_{\mathrm{i}}) = -\sqrt{2}\pi\varepsilon^{3/2}\frac{\sinh[\Omega Z(V_{\mathrm{i}})/2V_{\mathrm{i}}]}{\cosh(\Omega\pi/2V_{\mathrm{i}})} \tag{19a}$$

with

$$Z(V_{\mathrm{i}}) = \cos^{-1}\left(\frac{2V_{\mathrm{i}}^2 - \varepsilon}{2V_{\mathrm{i}}^2 + \varepsilon}\right). \tag{19b}$$

When the effect of the dissipative resistance on the inherent frequency is taken into account, the corrected inherent frequency is given by

$$\Omega = \sqrt{1 - \tfrac{1}{4}(\varepsilon^2 + a^2)}. \tag{6a}$$

The energy that an excited impurity mode transfers to the fluxon after the interaction of the fluxon with the excited impurity mode is given by

$$\Delta E = E_{\mathrm{im}} - \overline{E_{\mathrm{im}}}(V_{\mathrm{i}}, a_0, \theta_0) = -\frac{1}{\varepsilon}G(V_{\mathrm{i}})[2a_0\Omega\cos\theta_0 + G(V_{\mathrm{i}})]. \tag{20}$$

The critical amplitude a_{c} of the impurity mode above which the fluxon may escape is determined as

$$a_{\mathrm{c}} = -\frac{G(V_{\mathrm{i}})}{2\Omega\cos\theta_0}. \tag{21}$$

Figure 2 gives a plot of a_{c} against V_{i} for $\theta_0 = 0$. Figure 3 shows that, if $\theta_0 = 0$ and $a_0 = 1$, $\Delta E > 0$ for any initial velocity V_{i}. Therefore, the fluxon may, after the fluxon-excited impurity mode interaction, gain oscillating energy from the excited impurity mode and then the motion of the fluxon is essentially changed, depending dramatically on the phase θ_0 of the impurity oscillation at the collision instant. We have, in the zeroth-order approximation, obtained the explicit results of the fluxon-excited impurity oscillation interaction which are of benefit to understanding the physical connotation of this interaction.

In the following, we analyse the problem through numerically solving equation (11) with the initial conditions

$$X(0) = -6 \tag{22a}$$

$$\dot{X}(0) = 0 \tag{22b}$$

$$a_1(0) = 0 \tag{22c}$$

$$\dot{a}_1(0) = 0 \tag{22d}$$

$$a_2(0) = 0 \tag{22e}$$

$$\dot{a}_2(0) = 0. \tag{22f}$$

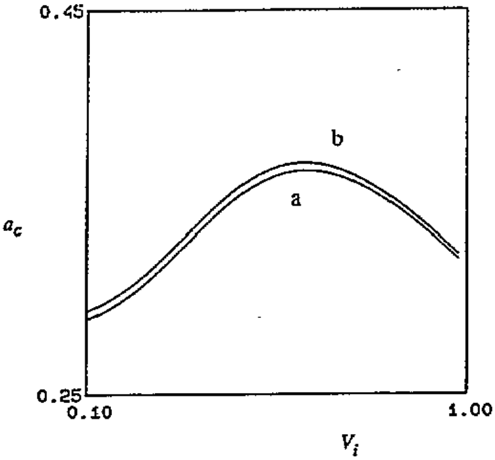

Figure 2. Dependence of the critical amplitude a_c on V_i for $\varepsilon = 0.7$ and different dissipative resistances: curve (a), for $\alpha = 0.013$; curve (b), for $\alpha = 0.2$.

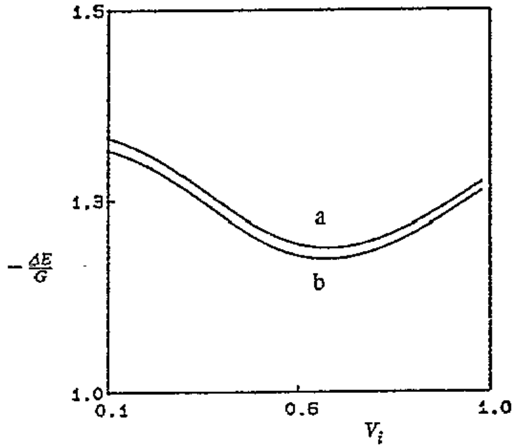

Figure 3. Dependence of energy exchange $-\Delta E/G$ on V_i for parameters $\varepsilon = 0.7$, $a_0 = 1$ and different dissipative resistances: curve (a), for $\alpha = 0.013$; curve (b), for $\alpha = 0.2$.

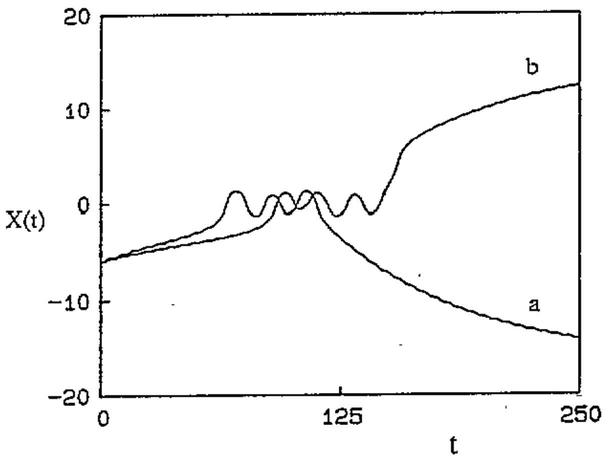

Figure 4. Numerical results of equation (11) as shown by the fluxon coordinate $X(t)$ versus time t for different A: curve (a), $A = 0.09$, the fluxon is reflected; curve (b), $A = 0.065$, the fluxon penetrates the microresistors.

The numerical results, as was expected, also indicate that the fluxon gains a part of the oscillating energy of the microresistors (the impurity modes) and finally escapes from the microresistors, as shown in figure 4.

We have, in terms of the semi-analytical approach mentioned above, drawn two important physical conclusions.

(1) A characteristic impurity mode can be excited and amplified by an AC drive.

(2) The fluxon can gain energy from the excited and amplified impurity modes (microresistors), then overcomes the potential wells of microresistors and the dissipative

resistance and finally escapes from the microresistors [4].

However, from equation (11a) we can see that the motion of the fluxon depends dramatically on the phase θ_0 of the impurity oscillation at the collision instant. If the fluxon can overcome the attractive potentials of microresistors and dissipative resistance, the total force exerted by the microresistors on the fluxon must be in the same direction (the x direction) when the fluxon is situated between the microresistors. The exertion strictly constrains the AC drive and the arrangement of the microresistors.

From now on, we turn our attention to solving equation (1a) numerically. We use a conservative numerical scheme to integrate equation (1a). Numerical calculations are performed in the spatial interval $(-70, 70)$ with discrete step size $\Delta x = 2\ \Delta t = 0.04$. For the Dirac δ function, we take its value to be equal to $1/\Delta x$ at $x = 0$, and zero elsewhere. When $A \ll 1$, fixed boundary conditions can be used: $\Phi(-70, t) = 0$ and $\Phi(70, t) = 2\pi$. The initial conditions are always taken as a fluxon centred at $X = -6$ with initial velocity $V_0 = 0$. Then we have

$$\Phi(x, 0) = 4\tan^{-1}[\exp(x + 6)] \tag{23a}$$

$$\Phi_t(x, 0) = 0. \tag{23b}$$

In order to perform numerical calculations, we have defined the centre $X(t)$ of the fluxon at which the field function $\Phi_k(x, t)$ is equal to π. The amplitude $a_i(t)$ ($i = 1, 2$) of the ith impurity mode is then given by

$$a_i(t) = \frac{\Phi_x(l_i - 0, t) - \Phi_x(l_i + 0, t)}{\varepsilon}. \tag{24}$$

The velocity of the fluxon is averaged over a period of 11 time units. The difference between the numerical results and the corresponding semi-analytical approach is introduced by making the small-parameter approximation and ignoring the radiation loss for the derivation of equation (11).

It can be seen that the fluxon overcomes the potential of the microresistors and escapes finally, as shown in figure 5. These are qualitatively consistent with the semi-analytical results for $A = 0.09$ and 0.0065, respectively (see figure 4). In addition, our calculation exhibits that there exists a critical value $A_c = 0.043$ (in the case of $\alpha = 0.013$) for the amplitude A of the AC drive. When the amplitude A of the AC drive is greater than A_c, the impurity modes are excited and amplified sufficiently that the fluxon can, through fluxon–microresistor interaction, be inelastically scattered by the microresistors and escape from the microresistors; on the contrary, the impurity modes cannot be amplified sufficiently and hence the fluxon is trapped and oscillates in the vicinity of the microresistors, as illustrated in figure 5. Figure 6 demonstrates that the impurity modes are excited and amplified after a finite time. It is also observed that the numerical results are in good agreement with the semi-analytical results.

In a real physical system the dissipative resistance of Josephson junctions is strong. In the next section we report our numerical simulation results in the strong dissipative resistance.

3. Strong dissipative fluxon in the Josephson junction

Under strong perturbation (α, $A < 1$ or $\ll 1$), although the perturbation theory mentioned above becomes invalid, it can be instructive to consider the dynamics problem of the

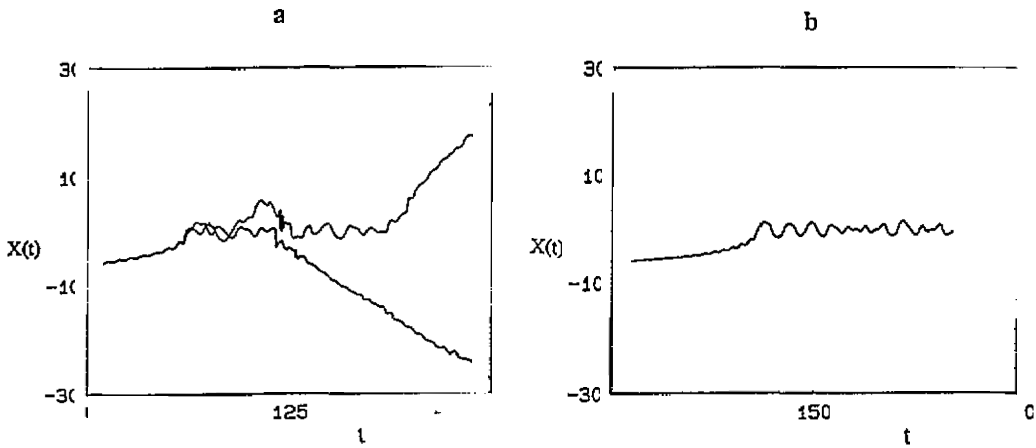

Figure 5. Numerical results of equation (1*a*), as shown by the fluxon coordinate $X(t)$ versus time t for different A: (a) $A = 0.09$ and 0.065, the fluxon is reflected and penetrates, respectively; (b) $A = 0.025$, the fluxon oscillates.

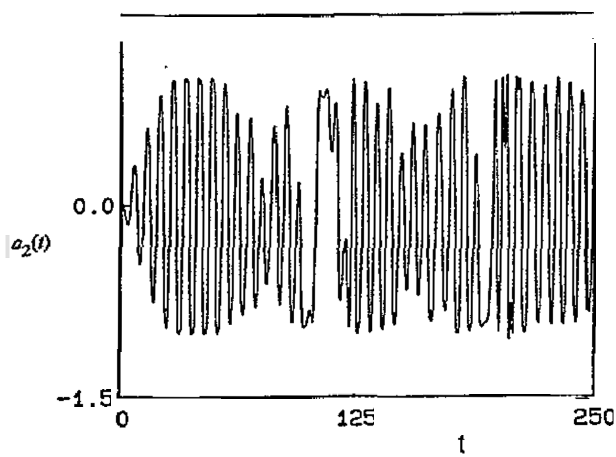

Figure 6. Numerical results of equation (1*a*) as shown by $a_2(t)$ versus time t for parameters $A = 0.065$, $\alpha = 0.013$ and $\varepsilon = 0.7$. Note that the impurity modes are excited and amplified after a finite time.

Josephson junctions with a strong dissipative resistance. Firstly we show that under strong perturbation (α, $A < 1$ or $\ll 1$) the fluxon is not destroyed, i.e. equation (1*a*) has a similar kink-type solution described by

$$\Phi(x,t) = \Phi_k + \Phi_{im}(x,t) + g(t) \tag{25}$$

where $g(t)$ is approximately given by

$$g(t) = \frac{A}{\sqrt{(\omega^2-1)^2 + a^2\omega^2}} \sin(\omega t + \varphi_0) \tag{26}$$

Here φ_0 is the initial phase.

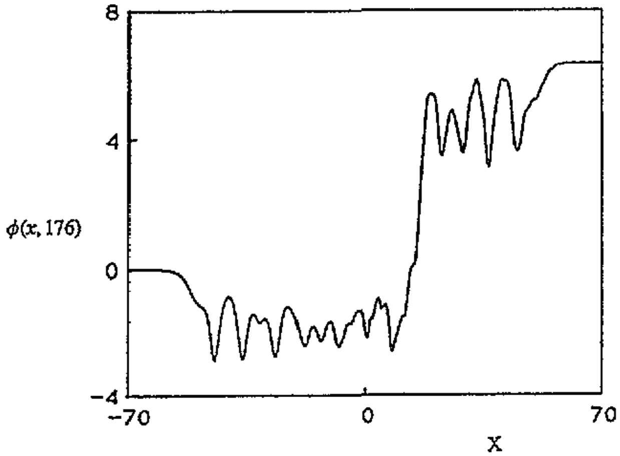

Figure 7. Numerical results $\Phi(x, 176)$ of equation (1a) for 15 microresistors, strong perturbations, $A = 0.8325$ and $\alpha = 0.23$. The results show that under the strong perturbations (α, $A < 1$ or $\ll 1$) the fluxon is not destroyed.

The numerical results of $\Phi(x, t_0)$ for a specific time t_0 are shown in figure 7. Now, let us demonstrate that the physical process stated in section 2 takes place similarly in the presence of strong dissipative resistance. That is, in the AC drive the impurity modes can be excited and amplified; when microresistors are properly arranged, through fluxon–impurity mode interaction the fluxon gains kinetic energy and then overcomes the potential wells of microresistors and the strong resistance to escape from the microresistors finally. In the following, we numerically solve equation (1a). To eliminate the boundary effects due to the AC drive ($A < 1$ or $\ll 1$), we take the AC drive as follows:

$$\eta_{\mathrm{AC}} = \begin{cases} A \sin(\omega t) & |x| \leqslant 50 \quad (27a) \\ A \exp[-5(x-50)] \sin(\omega t) & |x| > 50 \quad (27b) \\ A \exp[5(x+50)] \sin(\omega t) & |x| < -50. \quad (27c) \end{cases}$$

The following boundary conditions are used:

$$\Phi_x(-70, t) = \Phi_x(70, t) = 0 \tag{28}$$

The initial conditions are always taken as a fluxon centred at $X = -5$ and with initial velocity $V_0 = 0$. Then we have

$$\Phi(x, 0) = 4 \tan^{-1}(x + 5) \tag{29a}$$

$$\Phi_t(x, 0) = 0. \tag{29b}$$

As in section 2, we use a conservative numerical scheme to integrate equation (1a). Numerical calculations are performed in the spatial interval $(-70, 70)$ with discrete step size $\Delta x = 2\Delta t = 0.04$. We take $\alpha = 0.23$, $\varepsilon = 0.8$ and $n = 15$. The microresistors are arranged as follows:

$$l_1 = 0 \tag{30a}$$

$$l_{i+1} - l_i = 0.8. \tag{30b}$$

Similarly, for the Dirac $\delta(x, l_i)$ function, we take its value to be equal to $1/\Delta x$ at $X = l_i$, and zero elsewhere.

Firstly, we also find that there exists a critical value $A_c = 0.81$ for the amplitude A of the AC drive in a system in which the microresistors are properly (see equations (30)) arranged; when the amplitude A of the AC drive is greater than A_c, the fluxon gains kinetic energy from the excited and amplified impurity modes through fluxon–microresistor interaction while the microresistors lose their oscillating energy, so that the fluxon is inelastically scattered by the microresistors to escape from the microresistors finally, in the same way as for the case of the weak perturbations. The numerical results plotted in figure 8 show that the impurity modes can be excited and amplified in the strong dissipative resistance. The effects of microresistors on the propagation of a fluxon can be markedly changed because the impurity modes are excited and amplified by the AC drive before the fluxon–microresistor interaction. Figure 9 depicts that, for a specific arrangement of the microresistors when $A = 0.8325$, the fluxon overcomes the potential wells and the strong dissipative resistance and penetrates the microresistors; when $A = 0.9$, the fluxon is reflected. If the amplitude A of the AC drive is less than A_c, the fluxon is trapped and oscillates in the vicinity of the microresistors. Simultaneously, we show that, for the system of many microresistors, two essential conditions need to be satisfied so that fluxon can overcome the attractive potentials of microresistors and dissipative resistance. First, the amplitude of the AC drive must be large enough that the excited and amplified impurity modes have sufficient oscillating energy to give the fluxon. Second, interactions should depend dramatically on the phase θ_0 of the impurity oscillation at the collision instant.

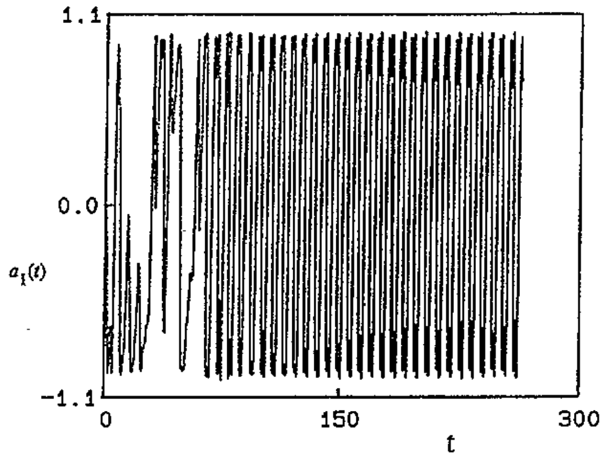

Figure 8. Numerical results $a_1(t)$ of equation (1*a*) for 15 microresistors and a strong dissipative resistance $\alpha = 0.23$: (a) $A = 0.90$; (b) $A = 0.8325$. Note that in a strong dissipative resistance the impurity modes can also be excited and amplified after a finite time.

The collision phase must be consistent at every collision. Thus the fluxon can gain oscillating energy from the microresistors and be accelerated in the same direction when the fluxon is between each pair of microresistors. The requirement of the consistency of the collision phase exerts a strict constraint on the motion of the fluxon, the initial phase of AC drive and the arrangement of microresistors. It is easy by only increasing the amplitude of the AC drive for a system consisting of a fluxon and microresistors to satisfy the first condition. It is much more difficult for the system to satisfy the collision-phase consistency, especially when the number of the microresistors is very large. Our results show only that an

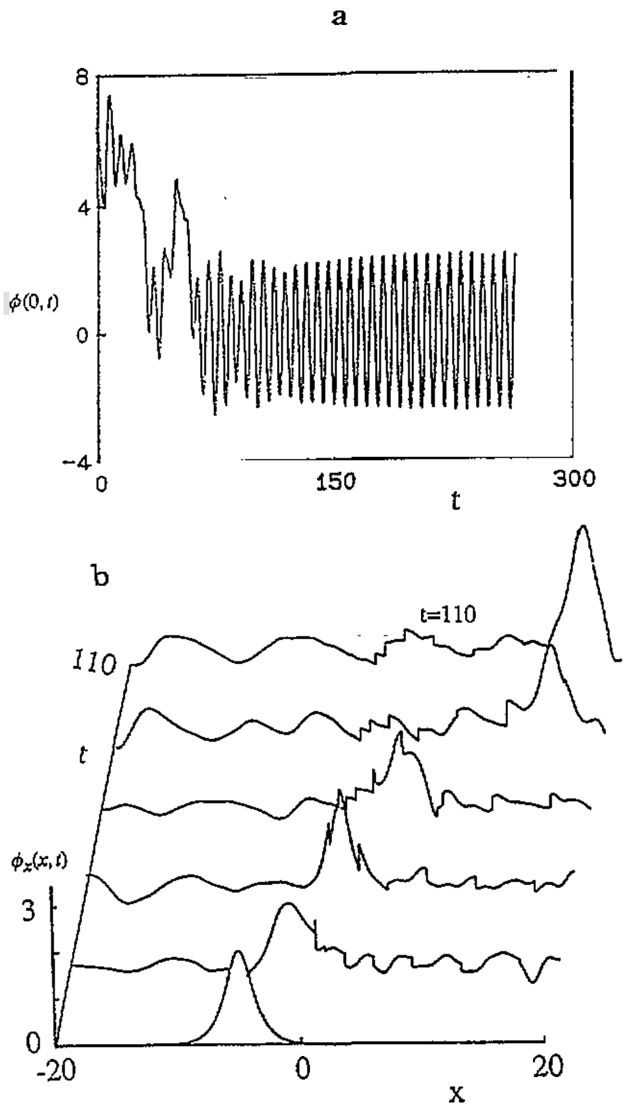

Figure 9. Numerical results of equation (1*a*) for 15 microresistors and the parameters $A = 0.8325$, $\alpha = 0.23$ and $\varepsilon = 0.8$: (a) $\Phi(0,t)$ versus time t; (b) $\Phi_x(x,t)$. Note that after the fluxon–microresistor interaction the fluxon penetrates the microresistors.

AC drive for a system consisting of a fluxon and microresistors may be found and can make the fluxon overcome the potential of the microresistor and dissipative resistance. However, because of the complex phase dependence, for a given system, it can only be verified through numerical simulation whether or not there is an AC drive which make fluxons escape from the microresistors.

4. Concluding remarks

In conclusion, we have considered the propagation of a fluxon in a randomly located inhomogeneous and resistant medium by the aid of perturbation theory and a numerical method. The motion of the fluxon in the randomly located inhomogeneous and resistant medium depends on the fluxon–microresistor interaction. Both the semi-analytical and the full numerical results show that in an AC drive the fluxon can overcome the attractive potentials of the microresistors and dissipative resistance and then escape from the microresistors. The threshold values of the AC drive allowing a fluxon to escape from the microresistors have been calculated. Finally, we give a brief physical explanation of this phenomenon as follows: the microresistors generate a new type of inherent localized structures, so-called impurity mode; the impurity mode can, even in the presence of a strong dissipative resistance, be excited and amplified by the AC drive; the fluxon gains, through the strong fluxon–microresistor interaction, kinetic energy to overcome the attractive potentials of the microresistors and the strong dissipative resistance and to escape from the microresistors finally.

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